of simultaneous equations at each frequency. Unfortunately, it is not always possible to uncouple the equations as the following example shows:

$$\mathbf{R}_0 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{R}_{\mathcal{L}} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \quad Z_C = \begin{bmatrix} 5^{1/2} & 2 \\ 2 & 5^{1/2} \end{bmatrix}. \tag{26}$$

Even if under certain cases it is possible, theoretically, to diagonalize the coefficient matrix in (24), such a transformation can easily lead to numerical instabilities unless it is of a numerically stable type such as an orthogonal transformation [6].

Although it is not always possible to obtain $\boldsymbol{\theta}$ in (25) in diagonal form, it is always possible to find a very stable transformation $\mathbf{M}$ such that $\boldsymbol{\theta}$ is in lower (or upper) Hessenberg form [6]. The lower Hessenberg form of $\boldsymbol{\theta}$ is such that a large number of the entries in $\boldsymbol{\theta}$ are zero, i.e., $[\boldsymbol{\theta}]_{ij} = 0$, $i = 1,\cdots,n-2$, and $j = (i+2),\cdots,n$. Thus the Hessenberg form is in "almost" lower triangular form.

Gaussian elimination utilizes row operations to reduce the coefficient matrix to lower triangular form and then back substitution is utilized to find the solutions. The majority of the operations are consumed in the reduction to lower triangular form.

With the transformation to Hessenberg form, (24) becomes

$$\begin{aligned}[\alpha \mathbf{I}_n + \beta \boldsymbol{\theta}]\mathbf{I}^*(0) &= \gamma \mathbf{M}^{-1}(\mathbf{R}_{\mathcal{L}} + \mathbf{R}_0)^{-1}[-\hat{\mathbf{V}}_s(\mathcal{L}) + \mathbf{R}_{\mathcal{L}}\hat{\mathbf{I}}_s(\mathcal{L})] \\ &\quad + \gamma \mathbf{M}^{-1}(\mathbf{R}_{\mathcal{L}} + \mathbf{R}_0)^{-1}\mathbf{E}_{\mathcal{L}} + \mathbf{M}^{-1}(\mathbf{R}_{\mathcal{L}} + \mathbf{R}_0)^{-1} \\ &\quad \cdot [\alpha \mathbf{I}_n + \beta \mathbf{R}_{\mathcal{L}} Z_C^{-1}]\mathbf{E}_0. \end{aligned} \tag{27}$$

Then one can employ row operations to reduce (27) to lower triangular form with back substitution being utilized to solve for the elements of $\mathbf{I}^*(0)$. $\mathbf{I}(0)$ can then be obtained from $\mathbf{I}(0) = \mathbf{M}\mathbf{I}^*(0)$.

Solving (24) with Gaussian elimination and back substitution requires on the order of $n^3/3$ per-frequency operations for large n. Solution via the reduction to Hessenberg form [solution of (27)] requires only $[n^2/2 + n/3]$ operations for triangularization, $[n^2/2 + n/2]$ operations for back substitution, and n^2 operations to form $\mathbf{I}(0) = \mathbf{M}\mathbf{I}^*(0)$ so that the total number of per-frequency operations has been reduced from on the order of $n^3/3$ with Gaussian elimination to $2n^2 + n$ for the Hessenberg reduction; a substantial savings for large n. Furthermore, the reduction to Hessenberg form is frequency independent and only needs to be performed once at the beginning of the frequency iteration.

If each line is connected to the reference conductor only through a single resistance (a very common situation), then $\mathbf{R}_0$ and $\mathbf{R}_{\mathcal{L}}$ will be diagonal and $(\mathbf{R}_0 + \mathbf{R}_{\mathcal{L}})^{-1}$ is trivial to obtain. $\mathbf{M}^{-1}$ is quite simple to obtain as a sequence of row operations [6] so that formation of (27) is not really so difficult.

Thus we are able to reduce the number of per-frequency operations in the homogeneous medium case from on the order of n^3 to on the order of n^2—a substantial savings for large n.

IV CONCLUSION

Numerically efficient methods of computing the frequency response of multiconductor transmission lines in homogeneous and inhomogeneous media illuminated by an EM field are presented. The formulations allow an efficient determination of the frequency response for cables consisting of a large number of coupled conductors with various port load conditions. The transformations used are numerically stable with respect to roundoff error and are frequency independent so that they need be determined only once at the beginning of the frequency iteration.

REFERENCES

[1] *Electromagnetic Pulse Handbook for Missiles and Aircraft in Flight*, Sandia Lab., Albuquerque, N. Mex., SC-M-710346, Sept. 1972.

[2] C. D. Taylor, R. S. Satterwhite, and C. W. Harrison, Jr., "The response of a terminated two-wire transmission line excited by a nonuniform electromagnetic field," *IEEE Trans. Antennas Propagat.*, vol. AP-13, pp. 987–989, Nov. 1965.

[3] C. R. Paul and J. C. Clements, "Coupled transmission lines," presented at the IEEE SOUTHEAST-CON, Louisville, Ky., Apr. 10–May 2, 1973.

[4] K. Ogata, *State Space Analysis of Control Systems*. Englewood Cliffs, N. J.: Prentice-Hall, 1965.

[5] C. R. Paul, "On uniform multimode transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 556–558, Aug. 1973.

[6] A. Ralston, *A First Course in Numerical Analysis*. New York: McGraw-Hill, 1965.

[7] R. L. Goldberg, "Focus on flexible flat cable and PCs," *Electron. Des.*, vol. 20, pp. 60–68, Dec. 7, 1972.

[8] W. T. Weeks, "Multiconductor transmission line theory in the TEM approximation," *IBM J. Res. Develop.*, pp. 604–611, Nov. 1972.

On the Calibration Process of Automatic Network Analyzer Systems

STIG REHNMARK

Abstract—**Formulas are presented for the direct calculation of the scattering parameters of a linear two-port, when it is measured by an imperfect network analyzer. Depending on the hardware configuration of the test set, the measurement system is represented by one of two flowgraph models. In both models presented, leakage paths are included. The error parameters, i.e., the scattering parameters of the measuring system, are six respective ten complex numbers for each frequency of interest. A calibration procedure, where measurements are made on standards, will determine the error parameters. One of many possible calibration procedures is briefly described. By using explicit formulas instead of iterative methods, the computing time for the correction of the scattering parameters of the unknown two-port is significantly reduced. The addition of leakage paths will only have a minor effect on computational complexity while measurement accuracy will increase.**

An important property of automatic network analyzers is that system errors can be brought to a minimum by a calibration process [1]. Two different measuring systems, represented by flowgraph models, will be considered in this short paper. Fig. 1 shows a schematic of the hardware configuration, with the digital computer excluded.

Which model to apply depends on whether the coaxial switch S_a is used or not. If the switch S_a is not included, the device under test has to be manually turned to be measured from both directions. In this case, the flowgraph model presented by Hand [2] is applicable. This model is shown in Fig. 2.

s_{11}, s_{12}, s_{21}, and s_{22} are the scattering parameters of the device under test. e_{00}–e_{32} are parameters representing errors in the system. By making measurements on standards, the error parameters can be determined. Three reflexion measurements with $s_{21} = 0$ are enough to determine e_{00}, e_{01}, and e_{11}. This can be done with a perfect termination, a direct and an offset short. A sliding load can simulate the perfect termination. A transmission measurement with $s_{21} = 0$ will give e_{30}. e_{22} and e_{32} can then be determined if $s_{21} = s_{12} = 1$ and $s_{11} = s_{22} = 0$, i.e., a through connection. A thorough description of the calibration process is given in [2]. Another similar calibration method is described in [3].

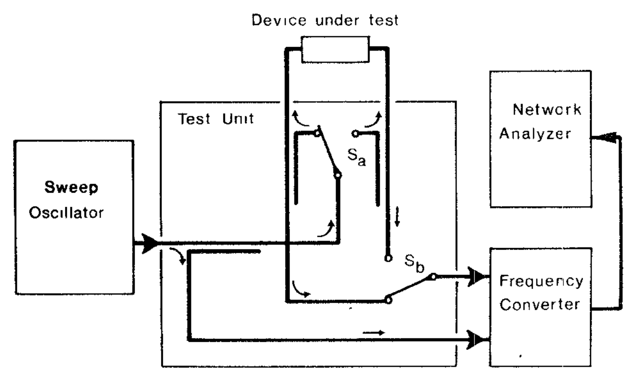

Fig. 1. Hardware configuration.

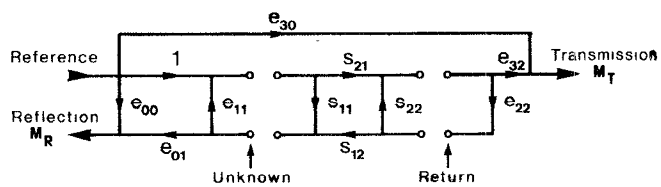

Fig. 2. Signal flowgraph of system model (switch S_a not included in test unit).

Manuscript received June 11, 1973; revised October 8, 1973.

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The reflected (M_{R1}) and transmitted (M_{T1}) signals are measured normalized to the reference signal. The unknown is then turned around to measure M_{R2} and M_{T2}. The scattering parameters of the device under test are given by the following equation system:

$$M_{R1} = e_{00} + \frac{s_{11}e_{01}(1 - s_{22}e_{22}) + s_{21}s_{12}e_{22}e_{01}}{D_1} \tag{1}$$

$$M_{T1} = e_{30} + \frac{s_{21}e_{32}}{D_1} \tag{2}$$

$$M_{T2} = e_{30} + \frac{s_{12}e_{32}}{D_2} \tag{3}$$

$$M_{R2} = e_{00} + \frac{s_{22}e_{01}(1 - s_{11}e_{22}) + s_{21}s_{12}e_{22}e_{01}}{D_2} \tag{4}$$

where

$$D_1 = 1 - s_{11}e_{11} - s_{22}e_{22} - s_{12}s_{21}e_{11}e_{22} + s_{22}s_{11}e_{11}e_{22} \tag{5}$$

$$D_2 = 1 - s_{22}e_{11} - s_{11}e_{22} - s_{12}s_{21}e_{11}e_{22} + s_{22}s_{11}e_{11}e_{22}. \tag{6}$$

As mentioned in [2], an iterative process can be used to find the scattering matrix of the unknown device. This has been found to be unnecessary since, by making substitutions in (1)–(4), the explicit solution has been found:

$$s_{11} = [C(1 + De_{11}) - ABe_{22}]/N \tag{7}$$

$$s_{12} = [1 + C(e_{11} - e_{22})]A/N \tag{8}$$

$$s_{21} = [1 + D(e_{11} - e_{22})]B/N \tag{9}$$

$$s_{22} = [D(1 + Ce_{11}) - ABe_{22}]/N \tag{10}$$

where

$$N = (1 + De_{11})(1 + Ce_{11}) - ABe_{22}^2 \tag{11}$$

$$A = (M_{T2} - e_{30})/e_{32} \tag{12}$$

$$B = (M_{T1} - e_{30})/e_{32} \tag{13}$$

$$C = (M_{R1} - e_{00})/e_{01} \tag{14}$$

$$D = (M_{R2} - e_{00})/e_{01}. \tag{15}$$

The scattering parameters of the unknown have to be determined at every frequency. Thus the explicit solution will save a lot of computer time.

When the coaxial switch S_a in Fig. 1 is included in the test unit, the measured device does not have to be manually turned during measurements. A flowgraph model for this system has been presented by Hackborn [1]. An explicit solution for the scattering parameters of the unknown has been found by Kruppa and Sodomsky [4].

In the flowgraph model by Hackborn, however, no leakage path is included. A flowgraph model including leakage paths is suggested in Fig. 3. Signals without parentheses apply when the switch S_a in Fig. 1 is in the left position, while signals in parentheses apply when the switch S_a is in the right position.

Again, by making measurements on standards, the error parameters r_{00}–r_{33} can be determined. The procedure will be similar to the one mentioned previously. The calibration process described by Hackborn [1] has to be extended to include two transmission measurements with $s_{12} = s_{21} = 0$ for the determination of r_{30} and r_{03}.

An analysis of the flowgraph yields

$$M_{R1} = r_{00} + \frac{s_{11}r_{01}r_{10}(1 - s_{22}r_{22}) + s_{21}s_{12}r_{22}r_{01}r_{10}}{D_3} \tag{16}$$

$$M_{T1} = r_{30} + \frac{s_{21}r_{32}r_{10}}{D_3} \tag{17}$$

$$M_{R2} = r_{33} + \frac{s_{22}r_{32}r_{23}(1 - s_{11}r_{11}) + s_{21}s_{12}r_{11}r_{32}r_{23}}{D_3} \tag{18}$$

$$M_{T2} = r_{03} + \frac{s_{12}r_{23}r_{01}}{D_3} \tag{19}$$

where

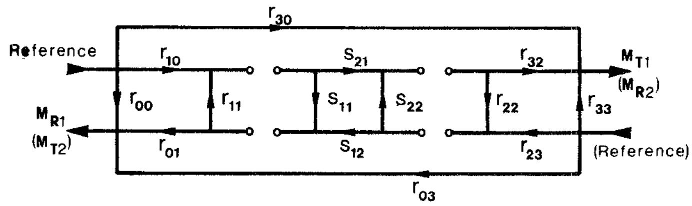

Fig. 3. Signal flowgraph of system model (switch S_a included in test unit).

$$D_3 = 1 - s_{11}r_{11} - s_{22}r_{22} - s_{12}s_{21}r_{11}r_{22} + s_{22}s_{11}r_{11}r_{22}. \tag{20}$$

The explicit solution to the scattering parameters of the measured device is then

$$s_{11} = [G(1 + r_{22}H) - r_{22}EF]/N_1 \tag{21}$$

$$s_{12} = E/N_1 \tag{22}$$

$$s_{21} = F/N_1 \tag{23}$$

$$s_{22} = [H(1 + r_{11}G) - r_{11}EF]/N_1 \tag{24}$$

where

$$N_1 = (1 + r_{11}G)(1 + r_{22}H) - r_{11}r_{22}EF \tag{25}$$

$$E = \frac{M_{T2} - r_{03}}{r_{01}r_{23}} \tag{26}$$

$$F = \frac{M_{T1} - r_{30}}{r_{10}r_{32}} \tag{27}$$

$$G = \frac{M_{R1} - r_{00}}{r_{01}r_{10}} \tag{28}$$

$$H = \frac{M_{R2} - r_{33}}{r_{23}r_{32}}. \tag{29}$$

As is seen above, the leakage paths will not significantly increase the computer time needed. Accuracy, however, will be improved.

ACKNOWLEDGMENT

The author wishes to thank Prof. E. F. Bolinder, Division of Network Theory, Chalmers University of Technology, Göteborg, Sweden, for his assistance and encouragement.

REFERENCES

[1] R. A. Hackborn, "An automatic network analyzer system," *Microwave J.*, pp. 45–52, May 1968.
[2] B. P. Hand, "Developing accuracy specifications for automatic network analyzer systems," *Hewlett-Packard J.*, pp. 16–19, Feb. 1970.
[3] H. V. Shurmer, "Calibration procedure for computer-corrected s parameter characterisation of devices mounted in microstrip," *Electron. Lett.*, pp. 323–324, July 12, 1973.
[4] W. Kruppa and K. F. Sodomsky, "An explicit solution for the scattering parameters of a linear two-port measured with an imperfect test set," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 122–123, Jan. 1971.

Design of Optimum Acoustic Surface Wave Delay Lines at Microwave Frequencies

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***Abstract*—Optimum procedures for designing microwave acoustic surface wave delay lines are given. Combined beam steering diffraction loss curves are provided as a function of the basic material parameter, the slope of the power flow angle, to allow optimum**

Manuscript received June 28, 1973; revised September 4, 1973.
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